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How can participation in a Learning Study foster prospective teachers' mathematical knowledge for teaching?

D. de Haan, S. de Vries, G. Roorda and P. Drijvers

Abstract This case study explores the impact of a Learning Study (LgS) course on pre-service teachers' (PSTs) Mathematical Knowledge for Teaching (MKT), and its central domains Specialized Content Knowledge (SCK), Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) in particular. Research suggests that PSTs may benefit from participating in a LgS – a Lesson Study guided by variation theory. However, little is known about how to integrate variation theory within the design of a LgS to foster PSTs' development. To address this issue, we designed and implemented a LgS course with two teams of seven PSTs in total. The research questions focus on whether the participants' MKT does develop, and which design elements foster this development. Data included interviews, written reflections and artefacts such as lesson plans. The results show that participants' MKT did develop, and that analyzing their students' pre-tests and observing of and reflecting on research lessons were most important in enhancing their KCS and KCT. Additionally, reasoning about (mis)conceptions was most helpful in increasing their SCK. However, not all design elements were as useful as expected. Further research might zoom in on enhancing the less effective parts of the LgS course.

Keywords Initial teacher education, Learning Study, Pre-service teachers, Mathematical Knowledge for Teaching

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1 Introduction

Teaching mathematics requires teachers to have Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008). MKT consists of both content knowledge and pedagogical content knowledge (Shulman, 1986). This means a deep understanding of mathematics, understanding how learners learn mathematics, and how to teach mathematics. Teachers usually start acquiring this knowledge in initial teacher education (ITE), where it is mostly offered as theory. In initial teacher education, it is often observed that pre-service teachers (PSTs) have difficulty with activating this theory-based knowledge when acting in the classroom practice. This is often attributed to the theory-practice gap in ITE, already described by Dewey in 1904 (Korthagen, 2010). Since Dewey, several studies examined and analyzed causes for this gap (e.g., Ball, 2000; Korthagen, 2010; Sugrue, 1997), which include the complexity of teaching (Ball, 2000; Korthagen, 2010) and the large impact of one's own experiences as a student in teaching and learning on PSTs' teaching practice (Korthagen, 2010; Sugrue, 1997). Therefore, teacher education programs have been looking for ways PSTs can develop MKT in a way that they manage to integrate the theoretical knowledge offered more easily into their classroom.

Lesson Study (LS) offers a possible solution for the described theory-practice gap, and its implementation in initial teacher education has initiated a growing amount of research in recent years (e.g., Bakker et al., 2022; Ni Shuilleabhain & Bjuland, 2019). LS is a cyclic process in which PSTs collaboratively design, conduct, observe, reflect on and replan lessons. When schools and ITE-institutes collaborate in such a way that PSTs can practice in school what they learn at ITE, LS can lead to synergy of practice and theory (Helgevold & Wilkins, 2020). However, Larssen et al. (2018) reported in their literature review of LS in ITE that they observed a lack of clarity in the definition of learning and in the use of learning theories. When teaching is not informed by theory, it can be minimized to building experience through imitation or sharing of best practices (e.g., McMahon et al., 2015). As a consequence, PSTs do not understand the relation between the quality of teaching and the learning of students (Nuthall, 2004). Therefore, in this study we chose a specific form of LS called Learning Study (LgS) (Lai & Lo-Fu, 2013; Lo, 2012; Xu & Pedder, 2015) that is built on a learning theory called variation theory. Variation theory is based on the principle that learning is made possible by distinguishing the critical aspects of a phenomenon. These aspects are seen against a background of targeted variations, which enable the learner to discern aspects of the learning object which have not been regarded as important before. This will be illustrated through examples in the next section.Variation theory needs deep content knowledge of phenomena to be taught, since the teacher has to know which critical aspects determine a certain phenomenon. Furthermore, variation theory also addresses knowledge of

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how students think, since the teacher has to know how the learners experience this phenomenon to determine how to use variation and invariance in teaching. Hence, variation theory brings theory, teaching and learning together. Therefore, LgS seems a promising approach to improve MKT of PSTs in initial mathematics teacher education.

In subjects other than mathematics, LgS “provides a powerful context for bringing theory and practice together” (Royea & Nicol, 2018, p. 14), and “appears to offer a way of increasing the speed at which trainees move to more complex ways of understanding teaching” (Davies & Dunnill, 2008, p. 15). These positive effects are confirmed by other studies: a case study of six PSTs in Chinese language (Ko, 2011) and an evaluation study of 341 second year PSTs (Cheng, 2014), both studies conducted at the Hong Kong Institute of Education. Three biology PSTs in a pilot LgS in Canada were also positive about their experiences to deepen their own learning (Tan, 2018). In primary mathematics education, a case study of a LgS course with 32 PSTs in Hong Kong fostered their understanding of the relationship between theory and practice (Lai & Lo-Fu, 2013), and therefore fostered their MKT. However, for now it is not evident which specific elements within the LgS generate processes that foster MKT. Furthermore, learning to use variation theory proved difficult for PSTs (Davies & Dunnill, 2008; Nicol & Royea, 2018; Tan, 2018).

The aim of this study is to identify in what way a LgS course in initial mathematics education in the Dutch context can foster PSTs Mathematical Knowledge for Teaching. To do so, we designed and implemented a LgS course, based on successful elements and processes derived from literature, taking into account any difficulties PSTs might have with applying variation theory.

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2 Theoretical background

2.1 Mathematical Knowledge for Teaching

Building on Schulman’s (1986) widespread and more general pedagogical content knowledge model, Ball et al. (2008) developed a specific model for mathematics: the Mathematical Knowledge for Teaching model. At the heart of the model are Specialised Content Knowledge (SCK), Knowledge of Content and Students (KCS), and Knowledge of Content and Teaching (KCT). Although Ball et al. (2008) identify more math-specific domains within their model, we focus on these three central domains because KCS and KCT are directly related to classroom practice, and SCK is a prerequisite for KCS and KCT.

Since the aim of our study is development of the knowledge domains, we have to look at these domains as processes that can be evolved. To do this, we use Skemp’s (1976) notions of relational and instrumental understanding as a lens. Where instrumental understanding refers to knowing the procedure, without knowing why it works, like following a recipe, relational understanding adds how it works and why. Relational understanding allows us to consider a concept from different perspectives, and to flexibly use this knowledge. We use this relational-instrumental lens within the model of MKT and its domains.

Specialised Content Knowledge (SCK) is the mathematical knowledge needed to teach, such as: being able to answer a "why" question; being able to link a topic you are explaining to the student's prior knowledge; being able to connect different representations. Where the student needs to develop math skills, the teacher needs to understand the core of why math procedures work to properly explain them to the student. The teacher needs to be able to give all kinds of examples, general and extreme, and non-examples, to explain mathematical concepts to students. Although it is possible for PSTs to understand mathematics instrumentally (and still teach), the goal however is for PSTs to develop relational understanding of mathematics, in line with the description of SCK.

Knowledge of Content and Students (KCS) is the ability of teachers to anticipate what students are thinking, what they will understand, what they will find confusing, whether they find something difficult. Central to this domain is the knowledge of students' conceptions and misconceptions about mathematical subjects, and where they come from. If this knowledge is based on experience, without knowing why and how these ideas originated, we call this instrumental understanding of KCS; if PSTs understand the relation between the (mis)conceptions and the nature of the mathematics underlying them, we call this relational understanding of KCS.

Knowledge of Content and Teaching (KCT) integrates knowledge of content and knowledge of teaching: what matters here is the design of the instruction. What order of examples do you use? Which representation do you start with? What are you doing now, what are you doing later? If the justification of the instructional design is based on "I do what the textbook suggests, because that works," we consider it instrumental understanding of KCT. If it is based on the PSTs' own relational understanding of the concept, we consider it relational understanding of KCT.

In the Netherlands, the knowledge base of the initial mathematics education program for secondary education explicitly references the Mathematical Knowledge for Teaching model (Ball et al., 2008) to indicate skills and competences mathematics teachers should master when entering teaching practice (Vereniging Hogescholen, 2017, p. 7). We therefore take this model as a starting point for the description of the required knowledge of the mathematics teacher, and consider PSTs' development in the knowledge domains of SCK, KCS and KCT as the intended outcome of our intervention.

In this study, we therefore mean by fostering PSTs' MKT: shifting towards relational understanding of SCK, KCS and KCT.

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2.2 Learning Study: Lesson Study with variation theory

To foster PSTs' MKT we will implement Learning Study. Learning Study (LgS) is, like Lesson Study (LS), a method where (pre-service) teachers collaborate on the cyclical design of a so-called research lesson, in which (pre-service) teachers' learning from their own students is central.

Characteristic for LgS, and different from LS, is the application of variation theory (Lo, 2012; Marton, 2015), a specific pedagogical learning theory which provides PSTs with tools to help their students to understand a particular concept, within any subject. Variation theory presumes that individuals see and understand phenomena from their own perspective (Xu & Pedder, 2015), and that learning takes place when the way of understanding a phenomenon, or Object of Learning, has changed (Lo, 2012; Marton & Booth, 1997). For this learning to occur, some aspects of the Object of Learning must vary while other aspects remain constant (Marton, 2015; Marton & Booth, 1997; Watson & Mason, 2006; Xu & Pedder, 2015). If a learner understands an Object of Learning in terms of its critical aspects, a learner has gained relational understanding of the Object of Learning (Marton, 2015, p. 14-15). To make this more concrete, the following three elements are critical in variation theory (Lai & Lo-Fu, 2013; Lo, 2012; Marton, 2015):

First, learning is always directed at something: the Object of Learning (OL). An OL "refers to what the students need to learn, to achieve the desired learning objectives" (Lo, 2012, p. 43). The OL is formulated in terms of critical aspects to be discerned. For instance, for the OL "recognizing a figure as a square," one critical aspect to be discerned is the number of sides. This should be four. Furthermore, also critical is that these sides have equal length and that there are four right angles. A non-critical aspect is the length of the sides, or the orientation of the square (whether it is rotated or not).

Second, (mis)conceptions of learners of the OL, in terms of (non-)critical aspects. It is important to have insight in how learners experience the OL, whether critical aspects are missing in their understanding of the OL, and whether they consider non-critical aspects as critical (Lo, 2012; Marton, 2015). For instance, if learners consider a rectangle to be a square, they do not see the equal length of the four sides as a critical aspect. Or, when they think a rotated square is not a square, they consider the way the square is drawn as critical.

Third, the design of the lesson, with use of systematic differences. Through carefully constructing a set of tasks in which (non-)critical aspects are systematically varied or kept invariant, learners are provided the opportunity to discern what is critical and what is not (Marton, 2015; Sun, 2011; Watson & Mason, 2006). For instance, for the OL "recognizing a figure as a square," learners first experience (standard) examples and non-examples of squares (like triangles, rectangles, circles, etc.), to discern what makes a square (its critical aspects are varied in the non-examples; non-critical aspects are kept the same

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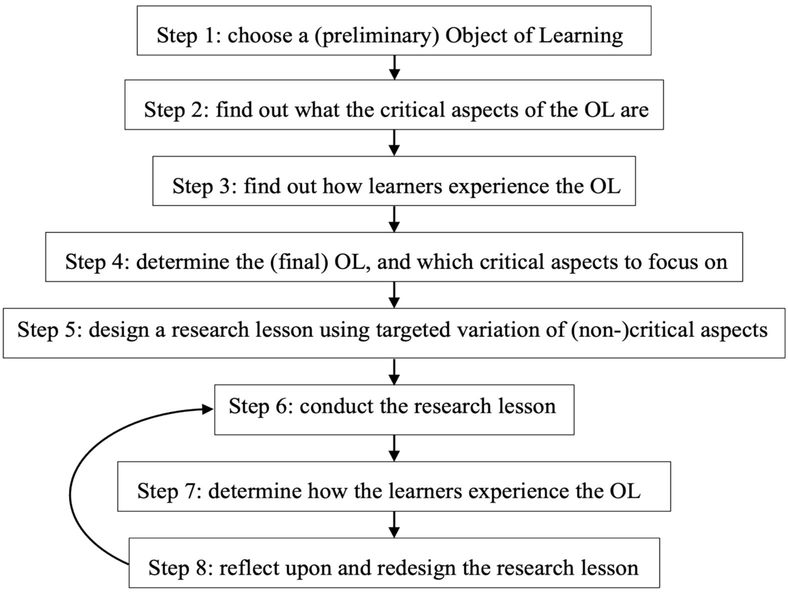
where possible). After that, they also need to see all kinds of different squares, standard and non-standard, with different sizes, different orientations, different colours, to be able to discern what is non-critical for a square to be a square (its critical aspects are now kept the same, its non-critical aspects are varied). In a LgS, these three elements of variation theory elaborated in eight steps form the back bone of the design (Lai & Lo-Fu, 2013; Lo, 2012; Xu & Pedder, 2015), see Figure 1.

Figure 1

Framework of a Learning Study



Note. Adapted from Lai and Lo-Fu (2013, p.79)

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2.3 Implementing Learning Study in initial teacher education

Research on Learning Studies provides indications of how to implement the steps described in Figure 1 for the case of initial teacher education.

First, for Steps 1 and 2, Cheng (2014) suggests to offer theory-based tutorials in which the framework of LgS and variation theory are presented, together with case study examples, as well as guidance on finding the OL and its critical aspects. The importance of the use of theory is confirmed by other research (Ko, 2011; Royea & Nicol, 2018). However, specifics of the content and pedagogy of the theory-based tutorials are not described. Regarding the guidance on finding the OL and its critical aspects (Step 2), Davies and Dunnill (2008) suggest to have PSTs design a kind of concept map in which the different critical aspects of a concept are included, and connections of (non-)critical aspects represent the way a PST understands the concept. This way of representing how a concept can be understood helps PSTs in being explicit about their own understanding of the OL. Since this concerns PSTs' own knowledge, we expect the concept map to play a role in fostering SCK.

For Steps 3 and 4, a common way suggested in literature is designing a pre-test, delivering and administering the pre-test in schoolpractice and analysing the results (Cheng, 2014; Lai & Lo-Fu, 2013; Ko, 2011). Because these steps lead to understanding learners' (mis)conceptions of the OL, we expect this to have an impact on fostering KCS. Based on the analysis, the PSTs choose their final OL (Lo, 2012).

For Steps 5-8, Cheng (2014) and Lo (2012) suggest to use case study examples and theory for the design. Since these steps are related to teaching issues, we expect them to influence fostering KCT. PSTs may have difficulties in applying Step 5, especially if variation theory is new to them (Cajkler & Wood, 2020; Davies & Dunnill, 2008; Royea & Nicol, 2018).

2.4 Research questions and conjecture map

The overarching research question of this study is: *How can a LgS course in mathematics education foster pre-service teachers' Specialized Content Knowledge, Knowledge of Content and Students and Knowledge of Content and Teaching?* Our overall conjecture is that fostering SCK, KCS and KCT of PSTs may benefit from conducting a LgS. In particular, we focus on two research questions: *RQ1: To what extent does participating in the LgS foster PSTs' development of SCK, KCS and KCT?* and *RQ2: Which design elements and mediating processes lead to fostering the participants' development of SCK, KCS and KCT?*

We used conjecture mapping (Sandoval, 2014) to conceptualize our design (see Figure 2). We reify the suggestions from research as described in the theoretical background into elements of the embodiment of the conjecture map. As a solution for the unfamiliarity with variation theory and its application, we added theory-based tutorials on relational and instrumental understanding to the embodiment. Within these tutorials, we use strategies offered by Kinach (2002). In repeatedly relating the "why" of using variation theory to the different steps within the LgS, we aimed for a comprehensive understanding of the use of variation theory.

We expect this embodiment to generate discussions in which the dialogue provides a mechanism for critical thinking and reasoning (Vrikki et al., 2017) about relational and instrumental understanding, and about variation theory; the embodiment will also generate pre-test analyses, lesson plans, etcetera. Fostering PSTs' SCK, KCS and KCT is the intended outcome of our intervention;

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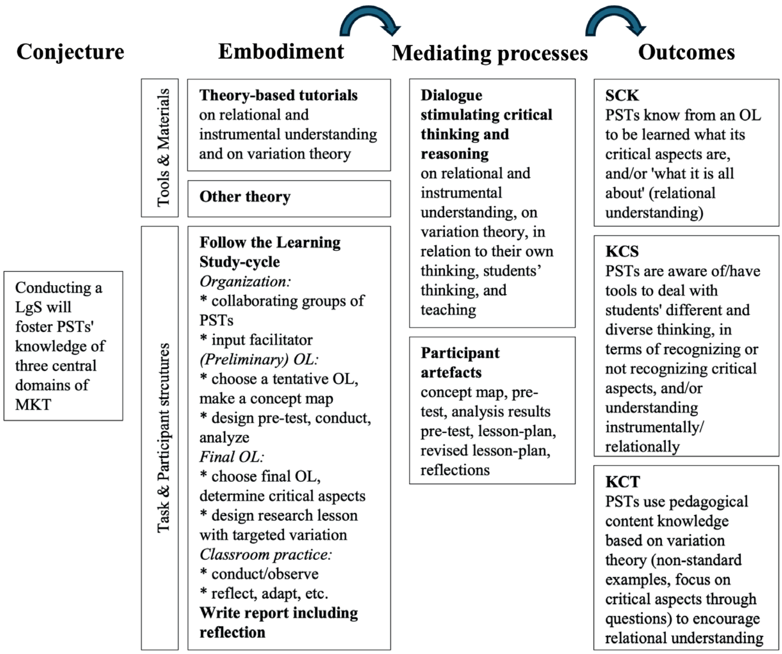
based on the theory, we can now look at each of the three domains with the lens of variation theory and with the lens of instrumental and relational understanding of the knowledge.

Figure 2

Conjecture map of the Learning Study course, based on Sandoval (2014)



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3 Method

The main question of this case study is design-oriented (Bakker, 2018). Sub-questions are related to outcomes and design principles. The aim of these evaluative questions is to obtain information about which parts of the embodiment seem effective to foster the development of PSTs' SCK, KCS and KCT.

3.1 Context

The teacher education program for teaching lower secondary education is a 4-year program, in which PSTs study mathematics, pedagogy, general and subject-specific methodology of teaching, and also do several internships.

One module in initial mathematics teacher education in the 4th (and final) year included LgS as part of the course content. The course took place from September 2021 - June 2022, on 15 Friday mornings, with an average of three hours per meeting. Two teacher educators taught the course, one of them in a dual role as teacher educator and first author of this paper. The other teacher educator observed and acted as a sounding board for the first author. The other days of the week, the PSTs were in their school-based practicum, in which they conducted the research lessons as part of the LgS.

3.2 Participants

Seven PSTs in two LgS teams, one of four PSTs (PST1-4) and one of three PSTs (PST5-7), participated in the LgS course. All PSTs were fulltime students in their last year of initial mathematics teacher education for lower secondary; all were between 21-24 years old.

3.3 Design and implementation of LgS course

The LgS course was structured based on the framework as presented in Figure 1. PSTs were asked to choose an OL within algebra, the largest domain within school mathematics. The implementation is presented in Table 1.

Table 1

Implementation of the Learning Study course, September 2021-June 2022

Step	When (2021-2022)	Clarification
Step 0: Learn about the goal of using variation theory (vt) through theory-based tutorials on relational and instrumental understanding (rel/inst) and on vt	September, October	Experience own rel/inst; Experience geometry-examples of use of vt
Step 1: Choose a (preliminary) Object of Learning	Early November	
Step 2: Find out what the critical aspects of the OL are	November	Design a concept map, with critical aspects, connections, structures, using the notions of rel/inst
Step 3: Find out how learners experience the OL	November - December	Design of pre-tests which were validated with help of their peers. For every task in the pre-test, the question was: “What do you know about the way the students understand the Object of Learning, when they do the task?”
	December-January	Conduct pre-tests in teaching practice
	January	Analyze pre-test results

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Step 4: Determine the (final) OL, and determine on which critical aspects to focus	February	Apply theory-based tutorials on how to design, with use of targeted variation in algebra. PST1-4 chose "ratio table" as OL, for Grade 7. PST5-7 chose "subtracting negative numbers" as OL, for Grade 7.
Step 5: Design a research lesson using targeted variation of (non-)critical aspects	February-March	Implement team-specific feedback on critical-aspects, on lesson-plans; facilitator providing other theory if appropriate
Step 6, 7 and 8: Conduct/observe, reflect and redesign	March - May	
Step 9: Reporting and reflecting	April-June	Process feedback on reporting.

3.4 Data collection and analysis

To answer the research question, we collected the following data:

- Interviews at the end of the course, in June 2022: six out of seven PSTs (PST1-6) were interviewed in a semi-structured way for 30-45 minutes from two perspectives: first, on what the elements of the LgS initiated for them, and second, on if and how their SCK, KCS and KCT were fostered. A visual aid containing all design elements of the LgS was presented to them during the interview;
- Individual written reflections of all seven PSTs (PST1-7), which were part of the final assessment of the course;
- Artefacts from both teams (pre-tests, pre-test analyses, lesson plans).

Interview transcripts and the individual written reflections were coded by analyzing the PSTs' answers on questions concerning outcomes of the LgS course (to answer RQ1) and on mentioning design elements and what these elements initiated for the PSTs (to answer RQ2). The coding was done using ATLAS.ti. The units of analysis were fragments in the interview in which aspects of the three main categories SCK, KCS or KCT became prominent, or aspects of the embodiment, or dialogue initiated by the embodiment, or dialogue leading to outcomes. Repetitions of information were combined as one fragment. All together, the fragments were roughly 80% of the interview and the written reflections. Talk about issues that had no link with teaching, learning, or the LgS course, was excluded. The first, second and third author independently coded the first three pages of two different interviews for design elements of the embodiment, mediating processes and outcomes. During a two-hour meeting, we elaborated on the design elements that caused different types of dialogue as mediating processes:

- reasoning about relational and instrumental understanding;
- reasoning about applying variation theory, by which we mean:

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- reasoning about the Object of Learning and its critical aspects,
- reasoning about (mis)conceptions of students or of themselves, and
- reasoning about teaching using systematic differences (in order for students to discern the critical aspects of the OL).

The mediating processes fostered SCK, KCS and KCT in different ways: in terms of relational and instrumental understanding, and in terms of variation theory. Although, as mentioned before, using variation theory might lead to relational understanding, we decided to code these outcomes separately, since in most fragments there was no link between the use of variation theory and relational understanding. The codes and subcodes were validated in a second two-hour meeting of the first three authors, after they had each independently coded a complete third interview. In the last step of coding, the first author checked the linkages between fragments within interviews and written reflections, as mentioned by each PST (fragments of embodiment, leading to mediating processes, leading to outcomes). These linked fragments resulted in individual narratives for all seven PSTs[1], that were checked by the second and third author. If PSTs refered to artefacts, for instance to a first version of the research lesson, the artefacts were checked to be able to interpret the fragments as intended. Next, the individual narratives were compared to look for patterns and see whether linkages between design, mediating processes and outcomes would exceed the individual level. If we found similar narratives for at least two PSTs, we distilled these as reasoning patterns.

To answer RQ1 on outcomes, the frequencies of codes SCK, KCS and KCT were calculated. Since some PSTs not only talked about their SCK, KCS and KCT with respect to the OL of their LgS, but also mentioned examples other than the OL, or talked in general terms about a knowledge-domain, in both interviews and written reflections, we refined the codes to capture the comprehensiveness of the knowledge-domains to “general”, “OL of the LgS” and “example other than OL of the LgS.”

To answer RQ2 on design elements of the embodiment and mediating processes, two perspectives were chosen: first, the co-occurrence in fragments of embodiment and mediating processes was checked, as well as the co-occurrence of mediating processes and outcomes and the co-occurrence of embodiment and outcomes. Second, reasoning patterns that occurred from similar individual narratives were described, using the conjecture map in Figure 2. How the embodiment linked to mediating processes and outcomes on a more overarching level, exceeding the individual narrative, was illustrated this way.

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4 Results

We will present the results per sub-question. For RQ1, we first present the number of fragments in the three global categories of SCK, KCS and KCT. Next, we distinguish between fragments related to the specific mathematical topic of the LgS and fragments related to other mathematical topics.

For RQ2 we first present the number of fragments of design elements and mediating processes that led to the outcomes. We then illustrate more general developments by presenting reasoning patterns within the LgS course.

4.1 The extent to which participating in LgS fosters the development of PSTs' SCK, KCS and KCT

In the interviews and written reflections, PSTs mentioned awareness of their own mathematical understanding (SCK), of their students' thinking (KCS), and of teaching issues (KCT).

Figure 3

Distribution of fragments over SCK, KCS and KCT (N=131)

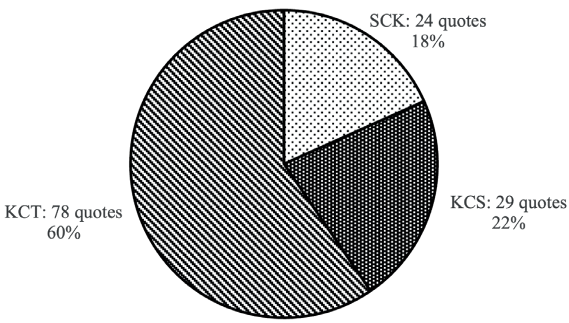

Figure 3 presents the overview of the number of fragments coded SCK, KCS and KCT. They concern all three knowledge domains, with a ratio of 3 : 1 : 1 (KCT - KCS - SCK).

Regarding the specified outcomes, as presented in Figure 4, most fragments on PSTs' development of SCK, KCS and KCT can be traced back to the teams' specified Object of Learning, as is shown in Figure 4: almost 60%. Of the remaining 40%, 2 out of 3 consist of general fragments, and 1 out of 3 are examples of specific topics within mathematics. There are no fragments on KCS on a specific topic other than the OL.

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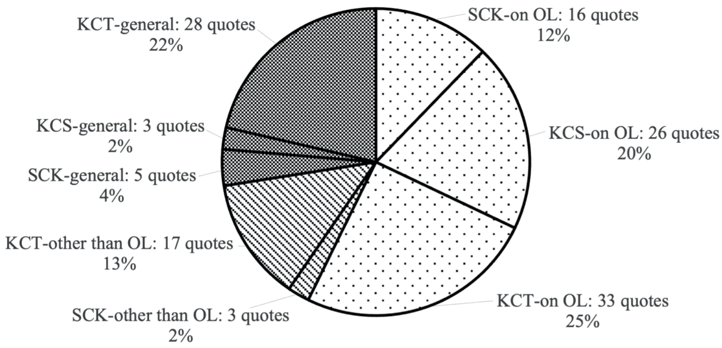
Figure 4

Distribution of fragments on OL and other general and specific topics (N=131)



Examples of fragments on the OL, of KCS, KCT and SCK, from different PSTs; PST2 had the concept of ratio as OL, PST6 and PST7 both had 'subtracting negative numbers' as OL:

> "a lot of students started thinking of those tables as ratio tables, you could just really see that in the worksheets."
> (PST2, KCS-OL)

> "I explained negative numbers to my students. I noticed that I was immediately a lot more aware of some pitfalls. For example, I dwelt longer on the concept of smaller/larger than."
> (PST7, KCT-OL)

> "I first thought, I don't understand, subtracting as calculating the difference on a number line, but then PST5 explained that really nicely to us, and then you can change your opinion on what works and why it works."
> (PST6, SCK-OL)

Examples of general fragments on KCS, KCT and SCK from different PSTs:

> "... trying to find out how the student thinks made me realize how they think, and that in turn helps me adapt a way of explaining accordingly."
> (PST4, KCS-general)

> "Much smaller steps! <I have to be aware> that I don't skip steps, because I expect them to be able to do what they can't do yet. I even noticed that the textbook skips quite a few steps."
> (PST5, KCT-general)

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> "I think variation theory can contribute to relational understanding by using different kinds of variations...because then it's just impossible to apply it more instrumentally."
> (PST2, SCK-general)

Fragments on specific topics within mathematics, other than the Object of Learning, were made mostly on KCT, by almost all PSTs. PST6, for instance, whose OL was on subtracting negative numbers, gave an example on explaining expanding expressions:

> "For example with expansion, with use of area, rather than a trick, you are more consciously working on 'why is it like this'."
> (PST6, KCT-other than OL)

For PST1, the OL was on ratio; however, this PST also gave an example on teaching quadrangles:

> "I had put a number of quadrangles on the board and I asked them 'How can you tell if something is a square or not?' And they quickly came to the two essential characteristics that it has 4 right angles and 4 sides with the same length."
> (PST1, KCT-other than OL).

Just two PSTs (PST2 and PST4) showed fragments on SCK, other than on the Object of Learning. An example from PST4, on recognizing a prism:

> "I remember doubting: is this a prism, or not? So then you start thinking about it yourself."
> (PST4, SCK-other than OL).

The results show that the LgS course contributes mostly to KCT, compared to KCS and SCK, and mostly to knowledge on the OL, compared to other subjects within mathematics.

4.2 Design elements and mediating processes leading to participants' development of SCK, KCS and KCT

Overviews of the co-occurrence of mediating processes and outcomes and of embodiment and outcomes for all PSTs are shown in Tables 2, 3 and 4. Reasoning patterns are described in Figures 5-10.

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Table 2

Frequencies of co-occurence for embodiment and mediating process categories

mediating process / **embodiment**	reasoning about rel/inst	reasoning about (mis) conceptions	reasoning about design of lesson with vt	reasoning about OL and critical aspects	**total**
theory-based tutorial rel/inst	4	1	2		7
theory-based tutorial vt		1	1	1	3
other theory			1	2	3
concept map		2		1	3
collaboration	2	6	2	1	11
input facilitator				1	1
analysing pre-test		9	1	2	12
observing students	2	3	2	1	8
design research lesson	1	1			2
reflect on research lesson	1	2	3	3	9
adapt research lesson		1	2		3
Total	10	26	14	12	62

From Table 2 we note that the analysis of the pre-test (Step 3 from Figure 1), collaboration, observing the students during the research lesson and reflecting on the research lesson (Steps 6-8 from Figure 1) are the parts of the embodiment that initiated most mediating processes, for all PSTs. Reasoning about (mis)conceptions is the mediating process that was generated the most.

Table 3

Frequencies of co-occurrence mediating processes and outcomes

outcome / ***mediating processes***	***SCK***	***KCS***	***KCT***	***total***
reasoning about rel/inst		3	6	9
reasoning about (mis)conceptions	3	7	5	15
reasoning about design of lesson with vt	1	1	6	8
reasoning about OL and critical aspects	4		5	9
total	8	11	22	41

Table 4

Frequencies of co-occurrence embodiment and outcomes

outcome **embodiment**	**SCK**	**KCS**	**KCT**	**total**
theory-based tutorial rel/inst	1		3	4
theory-based tutorial vt			2	2
other theory	1		2	3
concept map				
collaboration	1	2	3	6
input facilitator	1			1
analysing pre-test	1	5	1	7
observing students		1	2	3
design research lesson			1	1
reflect on research lesson			1	1
adapt research lesson				
total	5	8	15	28

The co-occurrence of the three types of codes (embodiment, mediating process and outcome) shows the same patterns for all PSTs in both mediating process – outcome, and embodiment – outcome, as shown in Tables 3 and 4: KCT is fostered the most, followed by KCS and SCK; analysing pre-test and collaboration are mentioned the most as design elements; reasoning about (mis) conceptions is the most often occurring mediating process.

For each PST, the linkage between embodiment - mediating process - outcome describes an individual narrative. Similar narratives are described as reasoning patterns. We determined six reasoning patterns, which turned out to be team-specific: three reasoning patterns within the team of PST1-4 with 'ratio' as their OL (presented in Figures 5-7), and three within the team of PST5-7 with 'subtracting negative numbers' as their OL (presented in Figures 8-10).

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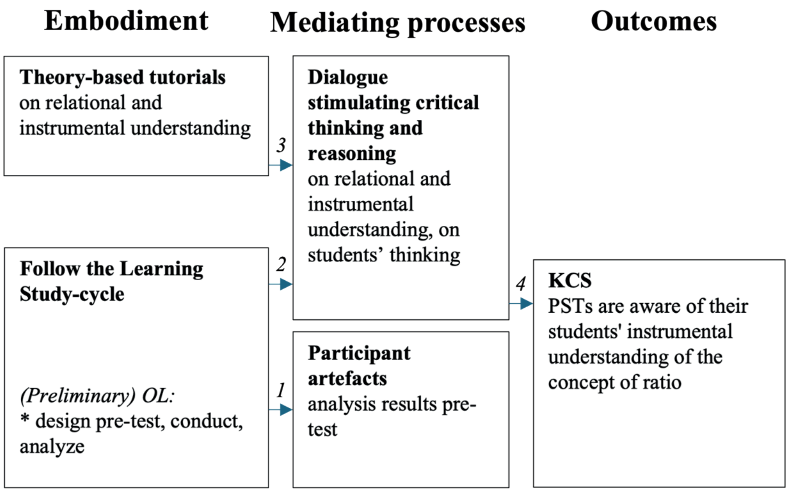
Figure 5

From pre-test to KCS, PST1-4



Fragments from PST1-4 show that analysis of the results of the pre-test about linear relationships (*1* in Figure 5) led to reasoning about how students fill in tables of linear, non-proportional relationships (*2* in Figure 5): when students use the context within the assignment, they can find the right answers; however, when they have to fill in a table, they wrongly treat the table as a ratio-table, despite their correct conclusion by reasoning from the context. The theory on instrumental and relational understanding provided the language to talk about how students understand the topic (*3* in Figure 5). This led to awareness of how students deal with tables of linear relationships (*4* in Figure 5). The following fragment describes the surprise of PST1, when analyzing the pre-test:

> "After we saw those results back, we thought: gosh. Actually a lot of students went wrong on this...."

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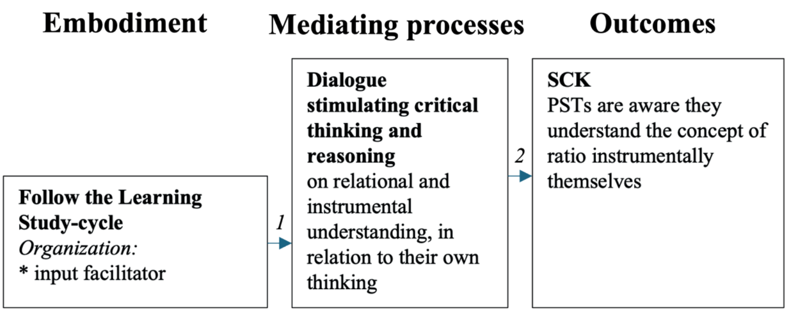
Figure 6

From input by teacher educator to SCK, PST1 and PST3



SCK was fostered through several parts of the embodiment. All PSTs were confronted with their own way of understanding the mathematical topic of their LgS. Input from the teacher educator (a task on filling in a table, which the PSTs wrongly considered to be a ratio table) was important for PST1 and PST3: they reflected on their own understanding (*1* in Figure 6) and concluded that they themselves showed the same instrumental understanding as their students (*2* in Figure 6). This reasoning pattern is illustrated by the following fragment:

> "When we were presented with a sample question, I made the same mistake I wanted to guard my students from, namely, immediately considering a half-filled table as a ratio table and filling in the blanks by doubling."
> (PST1)

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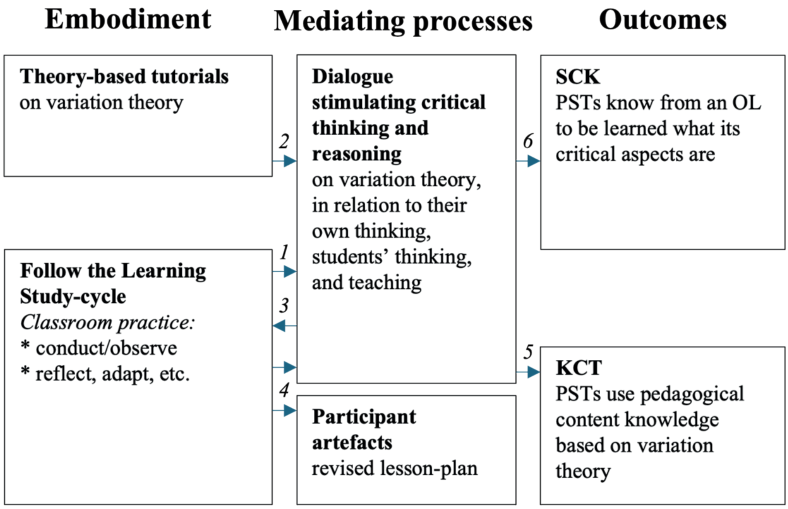
Figure 7

From reflecting on the research lesson to KCT and SCK, PST1-4



The OL and critical aspects chosen by PST1-4 (which was focused on ratio tables) were too limited for students to grasp the concept of ratio. This led to reflection on the research lesson, and to questioning its design principles (*1* in Figure 7). Studying literature on variation theory again and studying textbooks led to rethinking the OL and its critical aspects (*2* in Figure 7), which led to redesigning the research lesson, and conducting the lesson again (*3* and *4* in Figure 7). Reflecting on this process led to awareness of how to teach the concept of ratio using variation theory (*5* in Figure 7). This is illustrated by this fragment from PST4:

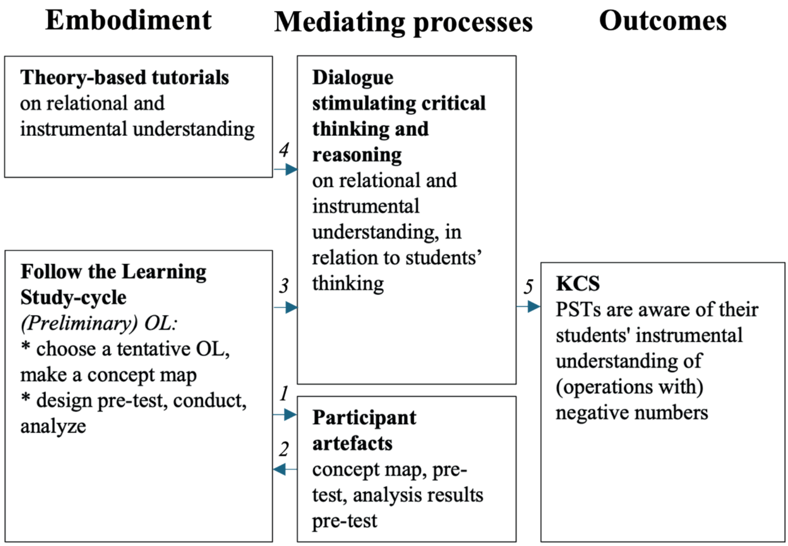
> "Then we started thinking, what is our goal? And my goal was that the students eventually know what ratio is, because if they know what ratio is, they will also be able to apply this in a table".

It also led to improved understanding of the critical aspects of the concept of ratio (SCK) for the PSTs (*6* in Figure 7), which is illustrated in the following fragment, from PST2, while teaching inverse proportionality in Grade 9:

> "One student asked me about proportionality: "That's just a linear formula that starts at zero, isn't it?" and then I thought: "That's exactly what we want!" And if I hadn't done this research, I wouldn't have consciously thought: "What a clever remark!", because I just knew that myself because of this research."

Figure 8

From concept-map to KCS, PST5-7



Fragments from PSTs 5-7 show that the design of a concept map on positive and negative numbers and all its operations led to an extensive pre-test. Analysis of the pre-test revealed many errors made by their students (*1* and *2* in Figure 8), which indicated an instrumental understanding of (operations with) numbers (*3* and *4* in Figure 8), as PST5 concluded in this fragment (*5* in Figure 8):

> "Especially the difference between two negative numbers, "two negative signs becomes plus", but that was also applied in other situations, for instance, −3−5=8, so they started overgeneralising rules."

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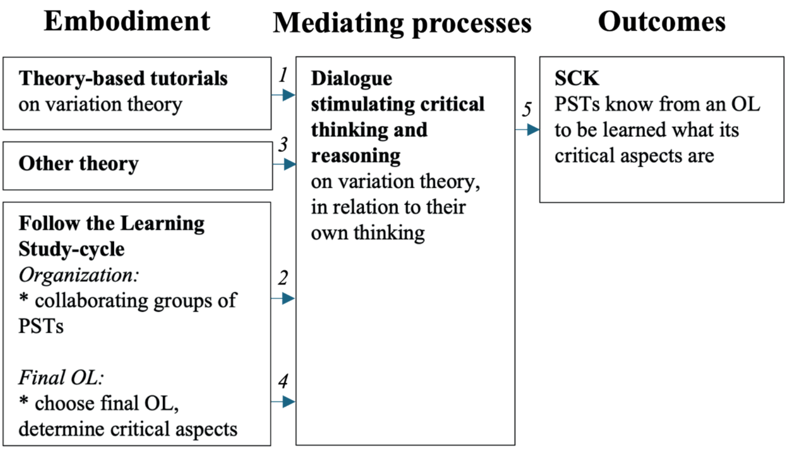
Figure 9

From collaborating on understanding theory to SCK, PST5-7



Kullberg et al. (2007) describe how variation theory can be used to promote subtraction with negative numbers by finding the difference of two numbers using the numberline. This idea was of great use for PST5-7 (*1* in Figure 9), since they needed each other to grasp the content properly to use it (*2* in Figure 9). Additionally, they read literature on process-object duality (*3* in Figure 9), which gave them insight on all critical aspects for this OL (*4* in Figure 9). This process fostered their SCK (*5* in Figure 9), as illustrated by this quotation from PST5:

> "The pre-test and theory <have yielded the essential aspects> and then we looked especially at Kullberg. Of course, negative numbers is a relatively easy subject for us, because we already understand it. And yet I did learn new things about how I understand it myself."

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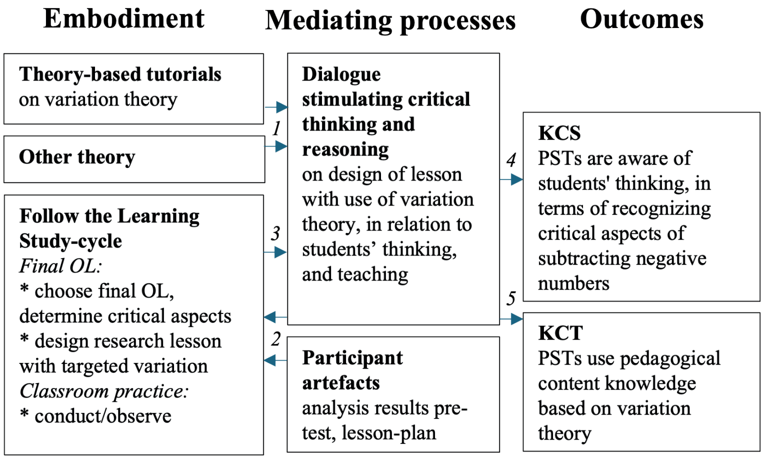
Figure 10

From results of the pre-test in the design of the lesson to KCS and KCT, PST5-7



The analysis of the results of the pre-test led to the choice of the final OL (which was subtracting negative numbers), with all necessary critical aspects (*1* in Figure 10). PST5-7 designed a research lesson in which all critical aspects were reviewed, using examples and non-examples (*2* in Figure 10). Their non-examples were mistakes from students, taken from the analysis of the pre-test. Their reflection on the research lesson (*3* in Figure 10) produced awareness that their students recognized the non-examples as their own thinking (*4* in Figure 10), and were able to adjust their understanding of operations with negative numbers (KCS), as shown in a fragment of PST6:

> "So, especially during the lesson, we actually found out that those non-examples are therefore quite helpful."

Discussing the design of the lesson also produced awareness how to teach negative numbers using variation theory (*5* in Figure 10), expressed by PST6 in the following way:

> "The way we set up the lesson, first number understanding, then getting acquainted with negative sign for subtraction, then the explanation with calculating the difference by distance on the number line, we noticed that it actually went quite well."

Tables 2-4 show us *what* happened; Figures 5-10 give us insight in *how* these processes took place, within the teams.

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5 Conclusion and discussion

In this case study, we investigated how a LgS course in mathematics education may foster pre-service teachers' SCK, KCS and KCT. In answering *RQ1 on the extent to which participating in the LgS fosters PSTs' development of SCK, KCS and KCT* we conclude that awareness, understanding and use of all three knowledge-domains have increased for all PSTs towards relational understanding – most for KCT, followed by KCS and SCK. For KCS and SCK, the increase is mainly restricted to the Object of Learning the PSTs chose within their team. KCT however was also used for other mathematical topics than the OL.

Concerning *RQ2 on the design elements and mediating processes that lead to fostering the participants' development of SCK, KCS and KCT* we conclude that the analysis of the pre-test, the whole teaching cycle of the research lesson (designing, conducting/observing, reflecting, adapting, etc.) and collaboration were elements of the design mentioned the most. The first two (analysis of the pre-test and the elements of the teaching cycle) met the expectation to have influence on fostering KCS and KCT respectively. The mediating process that occurred the most was reasoning about misconceptions – of PSTs themselves, leading to SCK, and of their students, leading to KCS and KCT.

From the reasoning patterns within the teams we conclude that the teaching cycle not only influenced KCT: it triggered thinking about the design of the lesson, and on the OL and its critical aspects, which led to an increase of knowledge on all three domains. Although PSTs' developments differed per team, this overall picture is the same for all PSTs.

Therefore, we conclude that elements of variation theory in the design of a LgS may foster PSTs' SCK, KCS and KCT– in addition to collaboration, which is a design element of every type of Lesson Study course. Hence, elements of variation theory do bring theory, teaching and learning together and thus provide the possibility to bridge the theory-practice gap, and from the reasoning patterns we see that this occurs through dynamic processes. However, not all elements of the embodiment of the LgS course proved successful in fostering MKT. First, the LgS course started with theory-based tutorials on relational and instrumental understanding, as suggested by Kinach (2002), to familiarize PSTs with variation theory. Through these tutorials, we aimed for comprehensive understanding of the use of variation theory. We found that the link between instrumental or relational understanding and critical aspects of the OL was not obvious for most PSTs. However, the input did provide a language to discuss the understanding of mathematical topics, especially when PSTs were not able to discuss the understanding in terms of critical aspects of the OL. Although relational and instrumental understanding remains very helpful in providing global language to talk about how a mathematical concept is understood, it is

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clear that the relation with the specific aspects of the OL that are considered critical by the learner (student or PST) is still difficult. A way to strengthen this relation (between critical aspects of an OL and relational understanding) is by using variation theory as a teaching strategy during the whole LgS course, for instance in presenting examples and non-examples of "what is a good question in a pre-test" and "what is not a good question in a pre-test", and finding critical aspects from these exercises: "teach as you preach" (Lunenberg, Korthagen & Swennen, 2007). Furthermore, PSTs can be encouraged to explore applying variation theory in geometry lessons during their teaching practice, as some of them already did, and which was considered much easier to apply than in algebra. Although applying variation theory in the design of the lesson was difficult, since a 'far-from-ideal' lesson was designed and conducted by a team of PSTs – yet an extensive increase of knowledge took place, because of the unexpected student reactions that resulted from it. In reflecting on such a lesson, it could be helpful to discuss this with language from variation theory (critical and non-critical aspects of the OL), as a deepening of the more global descriptions of relational and instrumental understanding.

A second element in the embodiment of the course that didn't work out as we expected was the concept map. Rather than playing a role in fostering SCK, this element of the design was not experienced as useful by most PSTs. It was hard for PSTs to come up with (non-)critical aspects of an OL. The goal of the concept map is to confront PSTs with their own knowledge – however, it was generally considered as less important for the whole process. To make this element more useful, the first draft of the concept map can be extended with input from the analysis of the pre-tests, and re-occur during the whole course as a reference. In this way, it may have more use for the fostering of knowledge. Also here, examples and non-examples of concept maps can be helpful for PSTs.

Concerning the specific fostering of SCK, we found that this occurred at times when students experienced some sort of conflict, or, as Mynott (2019) calls it: dissonance. For some PSTs, dissonance arose when they found out they made the same kind of mistake as their students (see Figure 6) and during the teaching cycle (see Figure 7); other PSTs experienced dissonance in their struggle in understanding an academic article (see Figure 9). This dissonance happened by chance, and was not part of the design. For a future design, it would be interesting to explicitly incorporate dissonance in the conjecture map, inspired by the model of potential teacher learning outcomes in lesson study as described by Mynott (2019).

Lastly, this case study has some limitations. First, because of the dual role of the teacher educator/first author, the interviews could have generated socially desirable responses, although the interviews took place after grading the PSTs. In a follow-up study, with another teacher educator teaching the course and facilitating the groups, the robustness of the design can be tested. Second, the

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interviews and written reflections were all self reports. In a follow-up study, audio recordings of PSTs collaborating could give more insight in occuring mediating processes. Third, the case study involves only seven PSTs. In a follow-up study with more PSTs, more patterns can be generated, which can give more insights in the relations between design elements, mediating processes and outcomes.

Despite these limitations, this study not only shows which elements of the design lead to the different outcomes; it also gives some insight in the dynamics of these processes through the reasoning patterns. Further study can build upon these results.

[1] PST7 was not interviewed - fragments of this PST are from the written reflection only

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Samenvatting

Hoe kan deelname aan een Learning Study bijdragen aan de vak- en vakdidactische kennis van leraren wiskunde in opleiding?

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In deze case study onderzoeken we hoe een Learning Study (LgS)-module bijdraagt aan de kennis van leraren wiskunde in opleiding (LIO's), met name aan de eigen wiskundekennis, de kennis over hoe leerlingen wiskunde leren en de kennis over hoe wiskunde te onderwijzen. Uit eerder onderzoek komt naar voren dat LgS (Lesson Study met variatietheorie) hiervoor goede mogelijkheden biedt, maar over hoe zo'n LgS er dan precies uit moet zien is minder bekend. We hebben een LgS-module ontworpen en met twee teams, in totaal zeven LIO's, uitgevoerd. De onderzoeksvragen richten zich op de ontwikkeling van de vak- en vakdidactische kennis van de LIO's, en op de onderdelen uit het ontwerp die deze ontwikkeling bevorderen. De data omvatten interviews, schriftelijke reflecties en artefacten zoals lesplannen. De resultaten laten zien dat de wiskundekennis van de LIO's toeneemt, met name door het redeneren over (mis)concepties, en dat het analyseren van de pre-tests van leerlingen en het observeren van en reflecteren op de onderzoekslessen het meest bijdragen aan de kennis over het leren en onderwijzen van wiskunde. Niet alle onderdelen uit het ontwerp werden als nuttig ervaren. Vervolgonderzoek kan zich richten op het verbeteren van de minder effectieve onderdelen van de LgS-module.

Kernwoorden lerarenopleiding, Learning Study, leraren-in-opleiding, wiskundeonderwijs

How can participation in a Learning Study foster prospective teachers' mathematical knowledge for teaching?

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